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Method of Calibrating Weights for Piston Gages

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Method of Calibrating Weights for Piston Gages

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Method of Calibrating Weights for Piston Gages

H. E. Almer

Generally weights for piston gages have odd denominations that are often not readily calibrated by intercomparison methods. Therefore, these weights are frequently calibrated by direct comparison methods. This paper presents direct comparison methods for calibrating piston gage weights for use with both equal-arm balances and single-pan balances. Methods of estimating the uncertainty of the values obtained are given. Also included are methods of checking for blunders or gross errors.

Keywords: Balance; buoyancy; calibration; standards; substitution weighing; transposition weighing; true mass; uncertainty; value.

1. Introduction

The method of calibrating pressure gage (piston gage) weights described herein is adequate for most purposes. The method employs simple weighing designs and includes using corrections for the standards and the buoyant effect of the atmosphere. Aside from blunders, such as misrecording the balance indication or the weights used, these account for the major sources of error in the mass value of the weights.

The comparisons may be made on any balance whose precision is adequate for the requirements of the weights being tested. If the precision of the balance is not known, the balance can be evaluated by tests such as described in ASTM's "Standard Methods of Testing Single-Arm Balances"[1]*.

Suitable mass standards, with established values or corrections, and appropriate uncertainties are required. For some purposes, the manufacturer's stated compliance with a class or other tolerance may be sufficient. In other cases it may be convenient to calibrate the built-in weights. Calibration procedures, such as are described in "Testing a Quick Weighing Balance" [2] or the ASTM's "Standard Methods of Testing Single-Arm Balances" [1] may be used. The various dial settings may be calibrated according to the method for calibrating dial-controlled weights described on pages 742/709 of "Testing a Quick Weighing Balance" [2].

Calibration procedures for use with both equal-arm balances and one-pan constant-load balances are described. For one-pan constant-load

*Figures in brackets indicate the literature references on page 25.

balances, both single and double substitution weighing methods using the built-in weights as standards, as well as substitution methods using known weights as standards, are described. For equal-arm balances, both the substitution weighing and the transposition methods of weighing are described. The computations are held to a minimum and will give the true mass value, M_m in the equation

$$F_e = \left[M_m\left(1 - \frac{\rho_a}{\rho_m}\right) + M_f\left(1 - \frac{\rho_a}{\rho_f}\right)\right]kg_L + \gamma C,$$

equation (10) of "Reduction of Data for Piston Gage Pressure Measurements" [3] as well as an appropriate estimate of the uncertainty of this value. Each weight is compared directly with a standard or group of standards of approximately the same mass as the weight being tested, by any of the methods described in Section 3. If the standards and the weights being tested are made of materials having approximately or nearly the same density, the errors due to buoyant effects are minimized. Each comparison will involve either duplicate measurements of the difference between the weight under test and the standard, as in double substitution weighing, or the same difference will be measured in each of the comparisons, as the sensitivity difference in single transposition weighing. These differences can be used to compute an estimate of the precision of measurement.

2. Calibration Procedures

The calibration of weights includes preparation of the weights for test, their comparison with standards, and data reduction. Each of these phases of the calibration is treated.

2.1 Preparation of Weights for Test

The weights are prepared for test as follows:

a. Unpack the weights carefully, being sure not to overlook any of the small weights which may be a part of the set.

b. Clean the weights by wiping with a soft cloth of nonabrasive material such as high quality cheesecloth. Solvents can be used to remove any foreign material that is not readily removed by wiping. Care must be exercised so that only foreign material is removed. Be certain that the solvents used will not injure the weights.

c. Place the weights and the standards in or near the balance so that they will come to temperature equilibrium before the weighings are made.

d. An information or summary sheet is prepared giving the following information: (See Sample Summary Sheet, Figure 1).

1. Identify the weights to be tested by the owner, set designation, and test number if one is used. The identification of the instrument of which the weights are a part should also be given.

2. The material of which the weights are made and their density.

3. List all of the weights in such a manner that, after calibration, the mass values found for them may be entered next to the appropriate weight.

4. Any other information needed to make a complete record of the test.

2.2 Comparison of Weights being Tested with Standards

Weighing Method - This suggested calibration procedure is based on the double substitution method of weighing (see Section 3) because it can be used with either one-pan constant-load balances or with equal-arm balances. Other weighing methods (Section 3) may be used where desirable or where the double substitution method is impracticable.

Standards - Generally the weights for piston gages have odd denominations not usually found in ordered sets of weights. Hence, the standard for a given weight may consist of more than one known weight. The buoyant effect of the atmosphere and associated errors can be minimized if, as far as practicable, the material of the known weights used for standards is of the same or nearly the same density as the material of which the weights being tested are made.

Sensitivity Weight - The mass of the sensitivity weight used depends in part on the on-scale range of the balance and in part on the mass differences between the weights being compared. The sensitivity weight should be as large as practicable. Its mass may be from one-fifth to one-half the on-scale range of the balance and it should be at least twice as large as the largest difference between the masses being compared. It may be necessary to trim one of the weights by adding small known weights to it to bring the difference between the masses to within these limits.

Comparison with Standard - Each weight of the set being calibrated is compared by the double substitution method (or other suitable methods) with "known" weights whose total mass is nominally equal to the mass of the weight being tested.

In sets having two or more weights of the same denomination, it is sometimes convenient to select one of the duplicate weights to be the "standard" for calibrating the other weights of the same denomination. This weight will be referred to as the "transfer standard". The "transfer standard" is calibrated with the "known" weight as mentioned above.

Environmental and Other Data Required - The date and time the work was done as well as the name of the observer should be entered on the observation sheet. The temperature, relative humidity, and barometric pressure at the time the weighings are made are also recorded on the observation sheet, so that buoyancy corrections can be computed and made, as required.

2.3 Data Reduction

Computation of Difference Between the Weight Being Calibrated and the Standard - The difference between the weight being calibrated and the standard is given by the difference in scale divisions between the balance indications for the weight under test and the standard. The method of computing this difference and of converting it from scale divisions to mass units is given for each method of weighing with the description of that method in Section 3. But for the following discussion, it will be assumed that the difference is in or has been converted to mass units.

The difference is expressed as follows:

$$(W_x - S) = a$$

where W_x is the weight under test

S is the standard; it may consist of one or more weights

"a" is the difference between W_x and S in mass units.

Application of Correction for Standard - The value of the standard may be expressed either as one number or as a nominal value and a correction, as follows:

Mass Value of Std. = Nom. Value + Correction

In general, the "Mass Value" of the standard means its True Mass value and its correction means its True Mass Correction unless otherwise indicated. The computation of the value of the weight being tested, W_x , is further described in Section 4.

When the value for S, whether expressed as one number or as a nominal value and a correction, is in terms of true mass, as it should be, the computed value of W_x will be its true mass value within certain limits which depend on the relative density of W_x and S (see Table 1). If S and W_x have the same or nearly the same density, the value found is for all practical purposes the true mass value. If the correction for S is known only in terms of apparent mass vs brass, as may be the case for some calibrated sets of standards, true mass corrections must be computed. This computation is as follows:

$$S_{TM} = S_{AM} + \rho_n(V_S - V_B)$$

where S_{AM} is the apparent mass vs brass value of the weight, that is the nominal value + the apparent mass correction.

S_{TM} is the true mass value of the weight

ρ_n is the density of normal air (1.2 mg/cm^3 at 20°C)

V_B is the volume of an equivalent mass of normal brass at 20°C*

V_S is the volume of the standard at 20°C.

The true mass correction is:

$$\text{Correction} = S_{TM} - \text{Nominal Value}$$

Buoyancy Correction - Buoyancy corrections are needed to take into account the difference in the buoyant effect of the air on weights of materials having different densities [4], especially where the densities of the materials are markedly different. It is a good practice to always compute (see Section 4), at least roughly, the magnitude of the correction to establish the order of magnitude with reference to the desired accuracy. If the correction is not significant it can be ignored.

If the volume of the weight is not known it may be computed from the mass and density relationship:

$$\text{Vol}_{W_x} = \frac{M_{W_x}}{D_{W_x}}$$

where M_{W_x} is the total mass of the weight, and

D_{W_x} is its density at 20°C.

Similarly the volume of the standard is:

$$\text{Vol}_S = \frac{M_S}{D_S}$$

where M_S is the mass of the standard, and

D_S is its density at 20°C.

*By definition the density of normal brass is 8.4 g/cm^3 at 0°C; that is approximately 8.3909 g/cm^3 at 20°C.

2.4 Uncertainty of Value of Weights

It is presumed that the weighings are being carried out by means of a measurement process whose parameters (precision, possible systematic errors, etc.) are known and sufficient evidence is collected to insure that the process is in a state of statistical control.* For each method of weighing there will be available a standard deviation to be associated with a single measurement of mass difference. This standard deviation will be based on considerable history and would be used in preference to a standard deviation based on the results of say one day's work. Such a value if available provides the means for judging whether or not to accept that day's measurements as being in control.

Uncertainty of Values Found for Weights - The uncertainty of the mass value of the weights consists of two parts; the uncertainty due to random errors of measurement and the systematic uncertainty due to the uncertainty in the value of the standard. The limit of the uncertainty due to random errors of measurement may be taken to be three times the standard deviation, σ , where σ is the standard deviation of the process. Therefore:

$$\text{Uncertainty of value} = 3\sigma + \text{uncertainty of standards.}$$

Uncertainty figures or statements are valid only if the measurement process was in a state of statistical control at the time the measurement to which the uncertainty figure applies was made. Therefore, every uncertainty figure associated with a mass value must be backed up by data adequate to determine that the weighing process was in a state of statistical control at the time the measurements used to determine that value were made.

*For a discussion of the procedures for estimating the parameters of the mass measurement process and for maintaining surveillance of the measurement process, see NBS Technical Note 288, Measurement Philosophy of the Pilot Program for Mass Measurement, by P. E. Pontius; and NBS Monograph 103, Realistic Uncertainties and the Mass Measurement Process, An Illustrated Review, by P. E. Pontius and J. M. Cameron.

In many cases the allowable imprecision is so much greater than the expected performance of the mass measurement process, based on previous measurements, that the measurement error could not exceed the allowable imprecision without some evident malfunctioning of the balance. In those cases it may be assumed that the results of the measurements are adequate if the balance appears to be working properly; but a formal uncertainty statement would not be made.

2.5 Checks for Blunders or Gross Errors

It is usually desirable to make some kind of check for at least gross errors in the values being reported. This is especially true where the standard consists of several weights because a weight may be overlooked when recording those used.

Checks by Comparing Values - In sets having several weights of the same denomination, one form of check is to compare the values obtained for weights having the same nominal value. If the value for one of these duplicate weights is markedly different from the others, this may indicate that the value is incorrect. For weights such as the "transfer standard" used as the standard for other weights of the same denomination, of which repeat weighings are made, the agreement of the results of the repeated weighings constitutes an acceptable check on the value.

Check by Weighing Groups - This kind of check cannot be used with large weights whose mass is near the capacity of the largest balance available. It is useful for weights which can be divided into groups not exceeding the capacity of the available balances. The procedure is:

a. Divide the weights into convenient groups.

b. Compare each group with appropriate standards.

c. Compare the value found for the group with the sum of the values for the individual weights making up the group. They should agree within the uncertainty of the measurements.

It is necessary to use some judgment in dividing the weights into groups. Small weights should not be grouped with large weights.

3. Weighing Methods

Any one of the four weighing methods - direct, direct-reading, substitution, or transposition - may be used for calibrating piston gage weights. The weighing method used depends in part on the balances and weights available, in part on the requirements of the job at hand, and in part on the preference of the person performing the test. Measurements made by either the substitution or transposition weighing method require more effort, but are in general more precise than those made by either the direct or the direct-reading weighing methods. The direct and direct-reading weighing methods are generally used only where relatively imprecise measurements can be tolerated and where only the minimum effort possible is justified.

The following comparison of the direct weighing method with the single transposition weighing method and direct-reading method with the single substitution method gives an indication of the differences in precision. Any weighing requires at least two observations. A direct weighing requires a no-load reading and a load reading. If the standard deviation of the mass measurement process, for one observation, is "σ" (random component of the uncertainty is 3σ), then the standard deviation of two observations combined is $\sigma\sqrt{2}$ (random component of the uncertainty of the two observations combined is $3\sigma\sqrt{2}$). In addition there must be added the uncertainty due to the inequality of the arms. This may vary from a few parts in a million to one or two parts in ten thousand, depending on the condition and quality of the balance. In contrast to this is the transposition method which also requires two observations, but since both observations are measurements of the same thing, the difference between the unknown weight and the standard (see Section 3.5, page 13), the standard deviation of this difference is $\frac{\sigma}{\sqrt{2}}$ (the random component of the uncertainty of the difference is $\frac{3\sigma}{\sqrt{2}}$). In transposition weighing the inequality of the arms drops out and does not become a part of the uncertainty. Thus, if the same balance is used for both methods, the uncertainty of the direct weighing method is twice that of the transposition method plus the uncertainty due to the inequality of the arms.

Both a direct-reading weighing and a single substitution weighing require two observations; therefore the random portion of the uncertainty is the same for both methods. In the direct-reading method the masses of the built-in weights are considered to be equal their nominal mass values (if corrections are applied it is no longer a direct-reading device), and the reading scale is assumed to have a one to one ratio with the indicated mass units. In a substitution weighing the weight being calibrated is compared with weights whose mass values are known and the reading scale is calibrated as a part of the substitution weighing. The uncertainty of the direct-weighing method is greater than the uncertainty of the single substitution weighing method by the uncertainty due to considering the nominal value of the built-in weights to be their actual value plus the uncertainty due to any reading scale error.

3.1 Direct Weighing

The equal-arm balance is the only type of balance that it is practical to use for the direct weighing method. First read and record the no-load, also called the zero load, indication. Then place the weight being tested on one of the pans, say the left pan; put sufficient known weights on the other pan to just bring the balance to an equilibrium condition such that the indication is the same as the no-load indication. The total mass of the known weights is the mass of the weight being tested.

3.2 Direct-Reading Weighing

The direct-reading weighing method requires a balance that is capable of directly indicating the weight of the load on the load receiving element in mass units. Most one-pan balances meet this requirement. With no-load on the pan, adjust the balance so that it reads zero; place the weight being tested on the pan and by proper manipulation of the controls bring the balance into a condition of equilibrium; read and record the indication. Since this is a direct-reading balance, the indication is the mass of the weight being tested, within the capability of the balance.

3.3 Substitution Weighing - One-Pan Constant Load Balances

Two substitution weighing modes for one-pan constant load balances are described. One uses known weights from an ordered set of weights as standards; the other mode uses the balance's built-in weights as standards. The method of using the built-in weights for standards is the same in principle as that using the usual known weights for standards though it may appear to

be different. With this method the built-in weights, removed from the total load on the beam to bring the balance beam to a position of equilibrium when a weight is placed on the pan, are the standards for that weight. The dial settings indicate which weights have been removed from the load.

To avoid negative balance indications or having to change dial settings during a given weighing, it is convenient to place a small tare weight on the balance pan and leave it on during the entire series of weighings. A weight of about one-fifth the full scale may be used.

3.3.1 Single Substitution Weighing - Using Known Weights from an Ordered Set as Standards (see Figure 2a for example).

Observation 1 - Place the weight being tested on the balance pan along with the tare weight. Release the balance, read and record the indication. This indication is designated I_1.

Observation 2 - Remove the weight being tested, but not the tare weight; and put enough standards on the pan to bring the balance into approximately the same position of equilibrium as it had when the weight being tested was on the pan, without changing the dial setting. Read and record the indication. This indication is designated I_2.

Observation 3 - Place a sensitivity weight on the pan along with the standards and the tare weight, if any. Read and record the indication. This indication is designated I_3.

The difference between the weight under test, W_x, and the standards, S, is:

$$W_x - S = (I_1 - I_2)\frac{W_s}{I_3 - I_2} = a$$

where W_s is the mass of sensitivity weight and

"a" is the difference between W_x and S in mass units.

3.3.2 Double Substitution Weighing - Using Known Weights from an Ordered Set as Standards (see Figure 2b for example).- Indications I_1, I_2, and I_3 of the double substitution weighing are obtained from observations 1, 2, and 3 of the double substitution weighing in the same manner as the corresponding indications for single substitution weighing described above, Section 3.3.1. There is a fourth observation in double substitution weighing which is described immediately below.

Observation 4 - Remove the standards from the pan, but leave the sensitivity weight and tare weight on the pan. Put the weight being tested on the pan with the sensitivity weight and tare weight. Read and record the indication. This indication is designated I_4.

The difference between the weight under test, W_x, and the standards, S, is:

$$W_x - S = \frac{I_1 - I_2 + I_4 - I_3}{2} \frac{W_s}{I_3 - I_2} = a$$

where W_s is the mass of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

3.3.3 Single Substitution Weighing One-pan Constant Load Balances Using Built-in Weights for Standards. (See Figure 3a for example.)

Observation 1 - Place the weight being tested on the balance pan with the small tare weight, and set the dials at the appropriate settings for that weight. That is, the balance indication is "on scale" when it is in equilibrium. Release the balance, if in equilibrium, record the dial settings. The weights represented by the dial settings are the standards for the weighing. Read and record the balance indication. This indication is designated I_1.

Observation 2 - Remove the weight being tested from the pan, leaving the tare weight; set the dials at "0". Release the balance; read and record the indication. This is the reading with the standard on. This indication is designated I_2.

Observation 3 - Place a sensitivity weight on the pan. Read and record the indication. This indication is designated I_3.

The difference between the weight under test, W_x, and the standards, S, (built-in weights indicated by dial settings in observation 1) is:

$$W_x - S = (I_1 - I_2)\frac{W_s}{I_3 - I_2} = a$$

where W_s is the mass of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

3.3.4 Double Substitution Weighing One-Pan Constant Load Balances Using Built-in Weights for Standards. (See Figure 3b for example). - Indications I_1, I_2, and I_3 of the double substitution weighing are obtained from observations 1, 2, and 3 of the double substitution weighing in the same manner as the corresponding indications for the single substitution weighing described above, Section 3.3.3. There is a fourth observation in double substitution weighing which is described immediately below.

Observation 4 - Place the weight being tested on the pan along with the sensitivity weight and the tare weight. Set the dials to setting used in observation 1. (This in effect removes the standards from the load on the balance.) Read and record the indication. This indication is designated I_4.

The difference between the weight under test, W_x, and the standards, S, (the standards are the built-in weights indicated by the dial settings in observations 1 and 4 above) is:

$$W_x - S = \frac{I_1 - I_2 + I_4 - I_3}{2} \frac{W_s}{I_3 - I_2} = a$$

where W_s is the mass of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

3.4 Substitution Weighing - Equal-Arm Balances

When an equal-arm balance is used for substitution weighing the weights being tested and the standards are interchanged on one of the pans and a counterpoise weight placed on the other pan. The counterpoise weight should have approximately the same mass as the weight being tested so that the indicator will be "on scale" when the balance beam is in equilibrium.

For interchanging the weight being tested and the standard it is convenient to select the pan on which increasing increments of load give numerically increasing indications. When the reading-scale reads from left to right, smallest numbers at the left end, adding weight to the left pan increases the readings; conversely, when the reading-scale reads from right to left adding weight to right pan increases the reading.

3.4.1 Single Substitution Weighing - Equal-arm Balance. (See Figure 4a for example). Assume that increased load on the left pan gives a numerically larger reading.

Observation 1 - Place the weight being tested on the left pan and the counterweight on the right pan. Release the balance, read and record the indication. If the balance is undamped, turning points are read and the rest point computed from them. [3] This indication is designated I_1.

Observation 2 - Remove the weight being tested and place enough known weights, that is standards, on the pan to bring the balance beam to approximately the same position of equilibrium as in observation 1. Read and record the indication. This indication is designated I_2.

Observation 3 - Add the sensitivity weight to that pan which will bring the indicator towards or past the center of the reading-scale. This indication is designated I_3.

The difference between the weight being tested, W_x, and the standards, S, is:

$$W_x - S = (I_1 - I_2)\frac{W_s}{|I_3 - I_2|} = a$$

where W_s is the mass of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

The absolute value of the sensitivity deflection, $I_3 - I_2$ is used, that is without regard to sign, as indicated by the symbol $|I_3 - I_2|$.

3.4.2 Double Substitution Weighing - Equal-arm Balance (see Figure 4b for

Indications I_1, I_2, and I_3 of the double substitution weighing are obtained from observations 1, 2, and 3 of the double substitution weighing in the same manner as the corresponding indications for the single substitution weighing described above, Section 3.4.1. The fourth observation of this double substitution weighing is described immediately below.

Observation 4 - Remove the standards from the left pan but leave the sensitivity weight on pan. Put the weight being tested on the pan with the sensitivity weight. Read and record the indication. This indication is designated I_4.

The difference between the weight being tested, W_x, and the standards, S, is:

$$W_x - S = \frac{I_1 - I_2 + I_4 - I_3}{2} \frac{W_s}{|I_3 - I_2|} = a$$

where W_s is the mass of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

The absolute value of the sensitivity deflection, $I_3 - I_2$, is used.

3.5 Transposition Weighing

Transposition Weighing - Transposition weighing is done on two-pan equal-arm balances. When this method of weighing is used counterpoise or tare weights are not required. The weight being tested is placed on one pan and the standard on the other pan; a reading is taken, then the weights are transposed. Either single or double transposition weighing methods may be used.

3.5.1 Single Transposition Weighing - Equal-arm Balance (see Figure 5a for example).

Observation 1 - Place the weight being tested on one of the pans, say the left pan, and standards on the other pan, the right pan. Adjust the amount of known weight on the pan so that the indicator will be "on scale" when the balance beam is in equilibrium. When the balance beam is in equilibrium, read and record the indication. This indication is designated I_1.

Observation 2 - Remove the weights from their respective pans and transpose them to the other pan. That is, the weight that was on the left pan is put on the right pan and the weight on the right pan is put on the left pan. Read and record indication. This indication is designated I_2.

Observation 3 - Add the sensitivity weight on the pan that will cause the pointer to move towards or past the center of the reading scale. Read and record the indication. This indication is designated I_3.

The difference between the weight being tested, W_x, and the standard, S, is:

$$W_x - S = \frac{I_1 - I_2}{2} \frac{W_s}{|I_3 - I_2|}$$

where W_s is the mass value of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

The absolute value of the sensitivity deflection, $I_3 - I_2$, that is without regard to sign, is used. Whether the weight being tested is heavier or lighter than the standard can be ascertained from the direction of motion of the indicator or the rules described on pages 15 and 15.

3.5.2 Double Transposition Weighing - Equal-arm Balance (see Figure 5b for example). Indications I_1, I_2, and I_3 of the double transposition weighing are obtained from observations 1, 2, and 3 of the double transposition weighing in the same manner as the corresponding indications for the single transposition weighing described above, Section 3.5.1. The fourth observation of the double transposition weighing is described immediately below.

> Observation 4 - Remove the weights from their respective pans and transpose them. Leave the sensitivity weight on the same pan that it was in observation 3. This indication is designated I_4.

The difference between the weight being tested, W_x, and the standard, S, is:

$$W_x - S = \frac{I_1 - I_2 + I_4 - I_3}{4} \frac{W_s}{|I_3 - I_2|} = a$$

where W_s is the mass value of the sensitivity weight and

"a" is the difference between W_x and S in mass units.

The absolute value of the sensitivity deflection, $I_3 - I_2$, that is without regard to sign, is used. Whether the weight being tested is heavier or lighter than the standard can be ascertained from the direction of motion of the indicator or the rule described below may be applied.

3.5.3 Determining the sign of the difference between two weights being compared in a transposition weighing - To find the sign of the difference "a", between two weights being compared by transposition weighing, in the expression:

$$W - S = a$$

Consider the following.

Example 1:

Obs. No.	Load on Pans		Balance Indication
	Left	Right	
0	0	0	I_0
1	W	S	I_1
2	S	W	I_2
3	$S+W_s$	W	I_3

Observation 0 is the measurement of the no-load or zero load equilibrium position. Observation 1 is the weighing with the weights W and S on the pans as indicated. Observation 2 is the weighing with the weights transposed. Observation 3 is the sensitivity weighing. If we assume the

balance's zero load equilibrium position is constant and that the arms are equal, then:

$$W - S = I_1 - I_0 \tag{1}$$

is a measure of the difference between the weights W and S in scale divisions. Similarly

$$S - W = I_2 - I_0 \tag{2}$$

is also a measure of the difference between the weights W and S in scale divisions. We now have two measures of the difference between W and S. Combining both measurements, we have:

$$W - S = I_1 - I_0 \tag{1}$$

$$S - W = I_2 - I_0 \tag{2}$$

$$2W - 2S = I_1 - I_0 - I_2 + I_0 \quad \text{(Sub.(2)from(1)} \tag{3}$$

Since the I_0's cancel each other we can rewrite equation (3)

$$2W - 2S = I_1 - I_2$$

$$W - S = \frac{I_1 - I_2}{2} = a' \tag{4}$$

The expression $W - S = \frac{I_1 - I_2}{2}$ states the difference between the weights in scale divisions. Since I_0 drops out when the two halves of the transposition weighing are combined, it is not necessary to find the zero load equilibrium position, Observation No. 0. The difference between W and S in mass units is stated in equation (5)

$$W - S = \frac{I_1 - I_2}{2} \frac{W_s}{I_3 - I_4} = a \tag{5}$$

Therefore, the complete transposition weighing requires only the three weighings shown in the following example (Example 2).

Example 2:

Obs. No.	Load on Pans		Balance Indication
	Left	Right	
1	W	S	I_1
2	S	W	I_2
3	$S+W_s$	W	I_3

In Example 2, if we assume that the reading scale reads from left to right, then since the sensitivity weight was added to the left pan we know that the term $\frac{W_s}{I_3 - I_2}$ in the expression,

$$W - S = \frac{I_1 - I_2}{2} \frac{W_s}{I_3 - I_2} = a$$

has a plus sign because I_3 is numerically greater than I_2. Therefore the sign for the difference "a" will depend on whether I_1 or I_2 is greater; if I_1 is greater, the sign is plus, and if I_2 is greater, the sign is minus. From the sign for the difference "a", we can ascertain whether W or S is the heavier weight. When the sign is plus, we have:

$$W - S = a \quad \text{or} \quad W = S + a$$

and W is heavier than S.

When the sign is minus we have:

$$W - S = -a \quad \text{or} \quad W = S - a$$

and clearly S is heavier than W. Sometimes the sign for the difference is not obvious and it may help to have rules for determining whether the sign for the difference between the weights being compared is plus or minus.

There are two cases: Case I sensitivity weight added to load on left pan; Case II sensitivity weight added to load on right pan. In each case there are four possible combinations of plus and minus signs relating I_1, I_2, and I_3 of the transposition weighing in the computation of the difference "a" between the weights being compared. These possibilities together with the appropriate sign are outlined below.

I. Sensitivity weight added on left pan.

a. I_1 numerically greater than I_2, and I_3 numerically greater than I_2, the sign for the difference, "a" is plus (+).

b. I_1 numerically greater than I_2, and I_3 numerically smaller than I_2, the sign for the difference, "a", is minus (-).

c. I_1 numerically smaller than I_2, but I_3 numerically greater than I_2, the sign for the difference, "a", is minus (-).

d. I_1 numerically smaller than I_2, and I_3 numerically smaller than I_2, the sign for the difference, "a", is plus (+).

II. Sensitivity weight added on right pan.

a. I_1 numerically greater than I_2, and I_3 numerically greater than I_2, the sign for the difference, "a", is minus (-).

b. I_1 numerically greater than I_2, but I_3 numerically smaller than I_2, the sign for the difference "a", is plus (+).

c. I_1 numerically smaller than I_2, but I_3 numerically greater than I_2, the sign for the difference, "a", is plus (+).

d. I_1 numerically smaller than I_2, and I_3 numerically smaller than I_2, the sign for the difference, "a", is minus (-).

Stated symbolically

I. Sensitivity weight added on left pan

a. $I_1 > I_2$ and $I_3 > I_2$ sign for "a" plus (+)

b. $I_1 > I_2$ and $I_3 < I_2$ sign for "a" minus (-)

c. $I_1 < I_2$ and $I_3 > I_2$ sign for "a" minus (-)

d. $I_1 < I_2$ and $I_3 < I_2$ sign for "a" plus (+)

II. Sensitivity weight added on right pan

a. $I_1 > I_2$ and $I_3 > I_2$ sign for "a" minus (-)

b. $I_1 > I_2$ and $I_3 < I_2$ sign for "a" plus (+)

c. $I_1 < I_2$ and $I_3 > I_2$ sign for "a" plus (+)

d. $I_1 < I_2$ and $I_3 < I_2$ sign for "a" minus (-)

4. Computation of Mass Value of Weight being Calibrated

In the preceding section, weighing methods and the method of computing the mass difference between the weight being tested were discussed and illustrated for each weighing method. The expression for this difference had the form $W_x - S = a$. But the required result, the mass value of the weight being calibrated, was not given. In the special case where the nominal value of the standard, S_n, may be considered its true mass value and no buoyancy correction is needed, then substituting for standard, S, its nominal value, S_n, in the expression:

$$W_x - S = a$$

we get $$W_x - S_n = a$$

and the mass value of $$W_x = S_n + a$$

The procedure is illustrated in figures 6 and 7.

4.1 Application of Correction for Standard and Buoyant Effect

In most cases, however, it is necessary to apply both a correction for the standard and to make a correction for the difference in the buoyant effect of the atmosphere on the weight being calibrated and on the standard. The expression $W_x - S = a$ gives the difference between the two weights in mass units under the conditions existing when the comparison was made. When the value of the standard is given as its nominal value, S_n, plus a correction, c, substitution for S its value, $S_n + c$, in the expression:

$$W_x - S = a \text{ (as computed)}$$

we get $$W_x - (S_n+c) = a \quad \text{(c is the correction for S from previous calibration)}$$

and $$W_x = S_n + c + a$$

This procedure is illustrated in figure 8.

When the correction, c, for the standards, S, is in terms of true mass the value found for W_x is its true mass within certain limits which depend on the relative densities of W_x and S. (See Table 1).

Where needed, the correction for the difference in the buoyant effect may be computed as follows:

$$(W_x - \rho Vol_{W_x}) - (S - \rho Vol_S) = a$$

$$(W_x - S) - a - \rho(Vol_S - Vol_{W_x})$$

or $$(W_x - S) = a + \rho(Vol_{W_x} - Vol_S)$$

where $\rho(Vol_{W_x} - Vol_S)$ is the buoyancy correction term, often expressed as $\rho\Delta V$

ρ is the air density at the time of comparison

a is the indicated difference between the weights

W_x is the weight being tested

S is the standard

The mass value of W_x when both the correction, c, for the standard, S, and the buoyancy correction, $\rho\Delta V$, are applied is computed as follows starting with the expression,

$$W_x - S = a \text{ (as computed)}$$

$$W_x - (S_n + c) = a \text{ (substituting for S its value } S_n + c)$$

plus buoyancy correction = $\rho\Delta V$

$$W_x = S_n + c + a + \rho\Delta V$$

The value for W_x is the true mass when the correction for S is in terms of true mass, within the appropriate uncertainty. This procedure is illustrated in figures 9 and 10.

4.2 Use of "Transfer Standard"

When one of the duplicate weights of a set having two or more weights of the same denomination is used as a "transfer standard" (Section 2), this weight is calibrated, using one of the weighing methods described in Section 3. The other weight or weights of the same denomination are then compared with the "transfer standard" by a suitable weighing method. When these comparisons have been completed, the "transfer standard" is recalibrated. The mean of the two values found for the "transfer standard" is the value used to establish the mass values of the weights for which it served as a standard. This procedure is illustrated in figure 11.

4.3 Added Weights

When the mass difference between weights being compared is larger than the on-scale range of the balance, it is necessary to add small weights to one or the other of the weights to get an on-scale balance indication. When an unknown weight is compared directly with a group of known weights, this is not a problem because only enough known weights are placed in use to bring the balance to the same equilibrium position as when the unknown weight was on the pan. However, it is sometimes more convenient to use one or more added small weights with the unknown weight so that a single large standard may be used.

In substitution weighing, using a one-pan balance, this is a straightforward procedure. For example, assume that the mass of the weight being calibrated is a little less than that of a convenient standard, as in example 1. (See Figure 12).

Example 1:

$O_1 \quad W_x + W_\Delta = I_1$

$O_2 \quad S = I_2$

$O_3 \quad S + W_s = I_3$

$W_x + W_\Delta - S = (I_1 - I_2) \frac{W_s}{I_3 - I_2} = a$

$W_x = S - W + a$ Solving for W_x

where W_x is the weight being calibrated

S is the standard

W_Δ represents the added known weight

W_s is the sensitivity weight

a is the difference between the masses

If in the above example S is two pounds and W_x is, say, 1.95 lbs, it is evident that this procedure is more convenient than if one had used standards equal to the mass of the weight being calibrated.

The use of an equal-arm balance permits greater flexibility because the added weights can be placed on either pan. But this adds complexity to the computations.

Consider the same kind of situation as in the example (Example 1) with the one-pan balance where the standard is heavier than the weight being calibrated and it is impractical to use a group standard, as would be the case when a "transfer standard" is used. If the difference is known prior to beginning the actual weighings of the calibration, then the necessary small weights can be placed on the pan with the weight being calibrated as shown below. (See Figure 13a).

Example 2:

Observation	Load on Pans		Indication
	Left	Right	
O_1	W_x+W_Δ	CW	I_1
O_2	S	CW	I_2
O_3	$S+W_s$	CW	I_3
O_4	W_x+W_Δ	CW	I_4

The difference is expressed as follows:

$$W_x + W_\Delta - S = \frac{I_1 - I_2 + I_4 - I_3}{2} \frac{W_S}{|I_3 + I_2|} = a$$

$$W_x = S - W_\Delta + a \qquad \text{Solving for } W_x$$

CW is the counterweight and the other symbols have the same meaning as above, for substitution weighing, with the one-pan balance.

But in many instances the magnitude of the difference between the weights will not be known before beginning the calibrations and the first half of the weighing will have been completed before it is apparent that small weights need to be added. In that case the most economical procedure is to place the added weight on the pan with the counterweight when the standard is on the other pan, as in Example 3. (See Figure 13b).

Example 3:

Observation	Load on Pans		Indication
	Left	Right	
O_1	W_x	CW	I_1
O_2	S	$CW+W_\Delta$	I_2
O_3	$S+W_S$	$CW+W_\Delta$	I_3

It is evident that the standard is heavier than W_x because additional weights were placed in the right pan to bring the balance to an on-scale equilibrium condition, when the standard was on the left pan. This is really the same situation as in Example 2 because, if the counterweight were increased by W_Δ, it would be necessary to add W_Δ to the load on the left pan in observation 1. The difference equation can be written in the form:

$$W_x - (S - W_\Delta) = (I_1 - I_2) \frac{W_S}{I_3 - I_2} = a$$

and

$$W_x = S - W_\Delta + a \qquad \text{Solving for } W_x.$$

This result is the same as that for Example 2.

The following argument will demonstrate that this is true:

Consider observation 1 as a measure of the difference between W_x and CW, and observation 2 as a measurement of the difference between S and $CW+W_\Delta$, then:

O_1	$W_x - CW$	$= kI_1$
O_2	$S - (CW + W_\Delta)$	$= kI_2$
$O_1 - O_2$	$W_x - CW - S + CW + W_\Delta$	$= k(I_1 - I_2)$

Since I_1 and I_2 are in scale divisions a factor, k, is applied to convert them into mass units. Generally k is equivalent to the term $\frac{W_s}{I_3 - I_2}$ in the foregoing examples. In this example

$$k = \frac{W_s}{|I_3 - I_2|}$$

$$W_x - S + W_\Delta = (I_1 - I_2) \frac{W_s}{|I_3 - I_2|} = a$$

$$W_x = S - W_\Delta + a \qquad \text{Solving for } W_x.$$

Consider the situation where the weight under test is heavier than the standard and the added weights were placed on the pan with the counterweight and the weight under test was placed on the other pan, as in Example 4. (See Figure 13c).

Example 4:

W_x heavier than the standard, S, and the counterweight, CW.

Observation	Load on Pans		Indication
	Left	Right	
O_1	W_x	$CW+W_\Delta$	I_1
O_2	S	CW	I_2
O_3	$S+W_s$	CW	I_3

Using the same argument as for Example 3

$$W_x - W_\Delta - S = (I_1 - I_2) \frac{W_s}{|I_3 - I_2|} = a$$

$$W_x = S + W_\Delta + a$$

Transposition Weighing - In transposition weighing added weights may be used during only part of the weighing or they may be used during the entire weighing, as required. Again, consider the case where the standard is heavier than the weights under test and the situation requires that the added weights be employed during the weighing, as in Example 5. (See Figure 14a).

Example 5:

Observation	Load on Pans		Indication
	Left	Right	
O_1	W_x+W_Δ	S	I_1
O_2	S	W_x+W_Δ	I_2
O_3	$S+W_s$	W_x+W_Δ	I_3

Compute as described for transposition weighing in Section 3.5.

$$W_x + W_\Delta - S = \frac{I_1 - I_2}{2} \frac{W_s}{|I_3 - I_2|} = a$$

$$W_x = S - W_\Delta + a \qquad \text{Solving for } W_x$$

This result is the same as that obtained for substitution methods.

Now consider the same case as illustrated in Example 5 except that it is necessary to employ the added weights during only one half of the transposition weighing, as in Example 6. (See Figure 14b).

Example 6:

	Load on Pans		
Observation	Left	Right	Indication
O_1	W_x+W_Δ	S	I_1
O_2	S	W_x	I_2
O_3	$S+W_s$	W_x	I_3

Treat each observation as a measurement of the difference between the weights.

O_1	$W_x + W_\Delta - S$	$= kI_1$
O_2	$S - W_x$	$= kI_2$
$O_1 - O_2$	$2W_x + W_\Delta - 2S$	$= k(I_1 - I_2)$

k is a factor to transform scale divisions into mass units.

$$2W_x - 2S = -W_\Delta + k(I - I)$$

$$W_x - S = \frac{-W_\Delta}{2} + \frac{k(I_1 - I_2)}{2} = a$$

$$W_x = S - \frac{W_\Delta}{2} + a$$

This result is similar to that obtained in the previous examples, the difference being that only half of the value of the added weights was applied.

Those of the above considerations that apply to substitution weighing are equally applicable to both single and double substitution weighing methods; and those that apply to transposition weighing are equally applicable to both single and double transposition weighing methods.

5. Air Density

In order to make the buoyancy correction computations (Section 4) it is necessary to know the density of the atmosphere in which the weighings were made. The density of the air may be computed in any of several ways [4][5]. Most of the methods given in the handbooks use tables which require either dew point measurement or wet and dry bulb thermometer readings. But, the approximate density of the air may be computed with sufficient accuracy (to about 10$\mu g/cm^3$) directly from temperature, relative humidity and barometric pressure by the formula given below [6] or by using the table (see Appendix) derived from that formula. The equation is:

$$\rho = \frac{0.46554P - R.H.(0.00252T - 0.020582)}{273.16 + T} \qquad [6]$$

where ρ = density of air in mg/cm^3

P = barometric pressure in mm

$R.H.$ = relative humidity in percent

T = temperature of air in degrees centigrade

The following example illustrates how the air density (ρ) is computed using this formula.

Compute the density of the air when:

The temperature is 24.8°C,
The relative humidity is 57%
The barometric pressure is 749.6 mm, Hg.

Substituting the temperature, relative humidity and barometric pressure for the symbols in the equation above

$$\rho = \frac{0.46554 \times 749.6 - 57(0.00252 \times 24.8 - 0.020582)}{273.16 + 24.8}$$

$$\rho = \frac{348.9689 - 57(0.041914)}{297.96}$$

$$\rho = \frac{346.6599}{297.96} = 1.163 \text{ mg/cm}^3$$

REFERENCES

[1] American Society for Testing Materials, 1968 Book of ASTM Standards, Part 30, Standard Methods of Testing Single-Arm Balances, ASTM Designation E-319-68.

[2] National Bureau of Standards Handbook 77, Volume III, Testing a Quick-Weighing Balance, pp 740/707 to 746/713.

[3] Cross, J. L.,Reduction of Data for Piston Gage Pressure Measurements, NBS Monograph 65.

[4] National Bureau of Standards Handbook 77, Volume III, Circular 3, pp 671/53 to 683/65.

[5] Handbook of Chemistry and Physics, Chemical Rubber Publishing Company, Cleveland, Ohio.

[6] Bowman, Horace A. and Schoonover, Randall M., Procedure for High Precision Density Determinations by Hydrostatic Weighing, NBS Journal of Research, Volume 71C (Engineering and Instrumentation) No. 3, 179-198 (1967).

FIGURE 1

WEIGHT SUMMARY SHEET

Test Performed For: X Y Z Corporation — Test No. 7561

Order No. Z-4035 — Date 10-23-67

Instrument Gage No. 71 — Observer C N A

Maker Doe Engineering Company — Set B

Inspected by: C N A — Date: 10-24-67 — Condition: New

Cleaned by: C N A — Date: 10-24-67 — Remarks:

Denomination	Material	Assumed Density
20 lb to 0.4246 lb	Stainless Steel	7.8 g/cm³ at 20°C
Piston Assembly	" "	7.8 g/cm³ at 20°C
Yoke Assembly	Composite	2.8 g/cm³ at 20°C

Weight Designation	Value	Weight Designation	Value
20 lb #361	19.99906 lb		
20 " #362	19.99998 "		
20 " #363	20.00016 "		
20 " #370	19.99950 "		
20 " #371	20.00016 "		
20 " #372	20.00020 "		
20 " #373	19.99943 "		
20 " #375	19.99789 "		
20 " #376	20.00020 "		
20 " #377	19.99997 "		
20 " #378	19.99973 "		
20 " #379	19.99991 "		
20 " #383	20.00032 "		
20 " #386	20.00023 "		
10 " #251	10.000657 "		
5 " #182	5.000425 "		
5 " #183	5.000363 "		
0.425 " #132	0.424620 "		
0.424 " #138	0.424637 "		
Piston Assy.	7.241383 "		
Yoke Assy.	2.334577 "		

Packed by: S. M. H. — Date: 11-1-67 — Report: ✓ — Date: 11-3-67

Shipped to: X Y Z Corporation — Via: R E A — Date: 11-7-67

FIGURE 2

Temperature 25.8 °C
Humidity 35 %
Barometer 752.6 mm
p =

FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

SUBSTITUTION WEIGHING
Single Pan Damped Balances
OBSERVATION SHEET

Sheet 1
Unit (Wts.)
Unit (Cc. & Dill.)

Observer PME | Balance M-4 | Date 11-10-66 | Set | NBS Test No. 177182

	Load	Dial Setting	Scale Reading	Computations
a	Wt X	590.4	I_1 29.24	X − Stds = (29.24 − 21.08) 20.01 / 41.10 − 21.08 = 8.16 × 20.01 / 20.02 = 8.16 mg = a
	L 500g L 10g L 50g AB 80 mg L 30g AB 100 mg	"	I_2 21.08	
	Stds + L 20, mg	"	I_3 41.10	This computation is usually shown as follows:
				X − Std = 8.16 × 20.01 / 20.02 = 8.16 mg
b	Wt Z	236.2	I_1 13.81	Z − Std = 13.81 − 20.57 + 33.82 − 40.60 × 20.01 / 40.60 − 20.57
	L 200g L 10g L 30g L 5g AB 200 mg	"	I_2 20.57	= (−6.76 − 6.78)/2 × 20.01 / 20.03
	Std + L 20 mg,	"	I_3 40.60	= −6.77 × 20.01 / 20.03 = −6.76 mg = a
	Wt Z + L 20 mg,	"	I_4 33.82	This computation is usually shown as follows:
				Z − Std −6.76 −6.78 Mean −6.77 × 20.01 / 20.03 = −6.76 mg

Single substitution weighing (a), and double substitution weighing (b), using known weights from an ordered set of weights as standards. The order in which the weighings are made and the method of computing the difference, in mass units, between the weight being tested and the standard are illustrated.

FIGURE 3

Temperature 23.1 °C
Humidity 52 %
Barometer 755.79 mm
p =

FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

SUBSTITUTION WEIGHING
Single Pan Damped Balances
OBSERVATION SHEET

Sheet 2
Unit (Wts.)
Unit (Cr. & Dill.)

Observer PCM | Balance M-4 | Date 9-17-64 | Set B | NBS Test No. 177064

	Load	Dial Setting	Scale Reading	Computations
a	TW* B2-138	24.0	I_1 23.52	Wt B2-138 - Stds. = (23.52 - 16.10) $\frac{20.01}{36.07-16.10}$ = 7.42 × $\frac{20.01}{19.97}$ = 7.44 mg = a
	TW 0	0.0	I_2 16.10	
	" TW + L 20 mg,	0.0	I_3 36.07	This computation is usually shown as follows:
		.	.	Wt B2-138 - Stds = 7.42 $\frac{20.01}{19.97}$ = 7.44 mg a
b	TW B3-160	130.6	I_1 24.93	B3-160 - Stds = $\frac{24.93 - 15.94 + 44.86 - 35.89}{2} \times \frac{20.01}{35.89-15.94}$ = $\frac{8.99+8.97}{2} \times \frac{20.01}{19.95}$ = 8.98 × $\frac{20.01}{19.95}$ = 9.01 mg a
	TW 0	0.0	I_2 15.94	
	" TW + L 20 mg,	0.0	I_3 35.89	This computation is usually shown as follows:
	TW B3-160 + L 20 mg,	130.6	I_4 44.86	B3-160 - Stds = $\frac{8.99}{8.97}$ mean 8.98 × $\frac{20.01}{19.95}$ = 9.01 mg a
		.	.	
		.	.	
		.	.	
		.	.	
	* TW indicates a tare weight was on the pan.	.	.	
		.	.	
		.	.	
		.	.	
		.	.	

Single substitution weighing (a) and double substitution weighing (b) using the balance's built-in weights as standards. The order in which the weighings are made and the method of computing the difference, in mass units, between the weight being tested and the standard are illustrated.

FIGURE 4

TEMPERATURE 24.8°C HUMIDITY 57% PRESSURE 749.6 TEST NO. 187 98/ SET c
OBSERVER PCN BALANCE B-1 DATE 8-9-86 SHEET 2 SERIES 1
PAN LOAD

11-10-65

LEFT	RIGHT				
		~~18.1~~	19.5	I_1	Wt # 251 - Std.
Wt # 251	Ctp 10.0005 lb	18.1	19.5		$= (37.6 - 40.4)\frac{100.0}{40.4 - 34.7}$
				37.6	$= -2.8 \times \frac{100.0}{5.7} = -49.1$ mb = a
X2 - 10 lb		~~18.9~~	21.4	I_2	
X2 - 500 mb	"	19.0	21.3		
a				40.4	
"	"	~~16.5~~	18.1	I_3	
		16.6	18.1		
	+100 mb			34.7	
Wt # 253	Ctp 5.0003 lb	~~19.5~~	25.2	I_1	Wt # 253 - Std
		19.6	25.1		$= \left(\frac{44.8 - 43.4 + 49.7 - 48.4}{2}\right)\frac{100.0}{48.4 - 43.4}$
				44.8	$= \frac{+1.4 + 1.3}{2} \times \frac{100.0}{5.0}$
X2 - 5 lb	"	~~19.8~~	23.6	I_2	$= +1.35 \times \frac{100.0}{5.0} = +27.0$ mb = a
X2 - 300 mb		19.9	23.5		
b				43.4	
"	"	~~21.7~~	26.7	I_3	
		21.8	26.5		
+100 mb				48.4	
Wt # 253 + X2 - 100 mb		~~22.3~~	27.1	I_4	
		22.5	27.3		
				49.7	

Single substitution weighing (a) and double substitution weighing (b) with equal arm balances, using known weights as standards. The order in which the weighings are made and the method of computing the difference, in mass units, between the weight being tested and the standard are illustrated.

FIGURE 5

TEMPERATURE 24.8 HUMIDITY 59.8 PRESSURE 749.6 TEST NO. 18790 SET B
OBSERVER PCM BALANCE B-1 DATE 8-9-66 SHEET 1 SERIES 2
PAN LOAD

11-10-65

	LEFT	RIGHT				
	Wt #183	X2 - 5 lb X2 - 0.3 lb	~~19.2~~ 19.2	21.7 21.6	I_1 40.8	Wt # 183 - Std $= \frac{40.8-41.5}{2} \times \frac{100.0}{\lvert 36.4-41.5 \rvert}$
	X2 - 5 lb X2 - 0.3 lb	Wt #183	~~19.9~~ 19.7	21.8 21.8	I_2 41.5	$= -0.35 \times \frac{100.0}{5.1} = -6.9$ mlb = a
a	"	" + X2 - 100 mlb	~~17.1~~ 17.2	19.3 19.2	I_3 36.4	
b	Wt #182	X2 - 5 lb X2 - 0.3 lb	~~19.4~~ 19.5	25.2 25.1	I_1 44.6	Wt # 182 - Std $= \frac{44.6-38.6+50.6-43.6}{4} \times \frac{100.0}{\lvert 43.6-38.6 \rvert}$ $= \frac{+6.0+7.0}{4} \times \frac{100.0}{5.0}$
	"	"	~~18.8~~ 18.8	19.8 19.8	I_2 38.6	$= +3.25 \times \frac{100.0}{5.0} = +6.5$ mlb = a
	" + X2 - 100 mlb	"	~~19.7~~ 19.7	23.9 23.8	I_3 43.6	
	Wt #182 + X2 - 100 mlb	X2 - 5 lb X2 - 0.3 lb	~~19.7~~ 19.8	30.8 30.7	I_4 50.6	

Single transposition weighing (a) and double transposition weighing (b). Known weights are used as standards. The order in which the weighings are made and the method of computing the difference between the weight being tested and the standard are illustrated.

FIGURE 6

Temperature 25.8°C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS		Sheet 1
Humidity 35%	SUBSTITUTION WEIGHING		Unit (Wts.)
Barometer 752.6	Single Pan Domped Balances		Unit (Cr. & Dill.)
ρ =	OBSERVATION SHEET		

Observer PMC	Balance M-4	Date 11-10-66	Set	NBS Test No. 177182

	Load	Dial Setting	Scale Reading	Computations
	Wt X	590.4	I_1 29.24	X - Stds
	L 500g L 10g L 50g AB 900mg L 30g AB 100mg	"	I_2 21.08	$= (29.24 - 21.08)\frac{20.01}{41.10 - 21.08}$
a	Stds. +L 20, mg	"	I_3 41.10	$= 8.16 \times \frac{20.01}{20.02} = 8.16$ mg = a This computation is usually shown as follows:
				X - Std = $8.16 \times \frac{20.01}{20.02}$ = 8.16 mg a
				Std = + 590.4 g
				X = 590.4 g + 8.16 mg
	Wt Z	236.2	I_1 13.81	Z - Std
	L 200g L 1g L 30g L 5g AB 200mg	"	I_2 20.57	$= \frac{13.81 - 20.57 + 33.82 - 40.60}{2} \times \frac{20.01}{40.60 - 20.57}$
	Std +L 20, mg	"	40.60	$= \frac{-6.76 - 6.78}{2} \times \frac{20.01}{20.03}$
b	Wt Z +L 20, mg	"	33.82	$= -6.77 \times \frac{20.01}{20.03} = -6.76$ mg = a This computation is usually shown as follows:
				Z - Std - 6.76 - 6.78 $-6.77 \times \frac{20.01}{20.03} = -6.76$ mg Std = + 236.2 g Z = 236.2 g - 6.76 mg

Single substitution weighing (a), and double substitution weighing (b), using known weights from an ordered set of weights as standards. The order in which the weighings are made and the method of computing the mass value of the weight under test when neither corrections for the standards nor buoyancy corrections are required.

FIGURE 7

Temperature 23.1°C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS		Sheet 2
Humidity 52 %	SUBSTITUTION WEIGHING		Unit (Wts.)
Barometer 755.79 mm	Single Pan Damped Balances		Unit (Cr. & Dill.)
ρ =	OBSERVATION SHEET		

Observer PCM | Balance M-4 | Date 9-17-64 | Set B | NBS Test No. 177064

Load	Dial Setting	Scale Reading	Computations
B2-138 *TW	24.0	I_1 23.52	Wt (B2-138) - Std. = (23.52 - 16.10) $\frac{20.01}{36.07-16.10}$ = $7.42 \times \frac{20.01}{19.97}$ = 7.44 mg = a
0 TW	0.0	I_2 16.10	
" TW +L20, mg	0.0	I_3 36.07	This computation is usually shown as follows:
			Wt (B2-138) - Std. = $7.42 \times \frac{20.01}{19.97}$ = 7.44 mg
			Std = 24.0 g
			Wt (B2-138) = 24.0 g + 7.44 g
B3-160 *TW	130.6	I_1 24.93	(B3-160) - Std. = $\frac{24.93-15.94+44.86-35.89}{2} \times \frac{20.01}{35.89-15.94}$
0 TW	0.0	I_2 15.94	= $\frac{8.99+8.97}{2} \times \frac{20.01}{19.95}$
" TW +L20, mg	0.0	I_3 35.89	= $8.98 \times \frac{20.01}{19.95}$ = 9.01 mg = a
B3-160 TW +L20, mg	130.6	I_4 44.86	This computation is usually shown as follows:
			(B3-160) - Std. = $\frac{8.99}{8.97} \times \frac{20.01}{19.95}$ = 9.01 mg
			Std = 130.6 g
			(B3-160) = 130.6 g + 9.01 mg
* TW indicates a tare weight was on the pan.			

Single substitution weighing (a), and double substitution weighing (b), using the balance's built-in weights as standards. The order in which the weighings are made and the method of computing the mass value of the weight under test, when neither corrections for the standards nor buoyancy corrections are required.

FIGURE 8

TEMPERATURE 24.8 HUMIDITY 57 PRESSURE 744.6 TEST NO. 1769/SET A
OBSERVER PCM BALANCE 0-1 DATE 1-9-67 SHEET 2 SERIES
PAN LOAD

11-10-65

LEFT	RIGHT			
Yoke Assy	Ctp 7.23 lb.	~~18.5~~	20.1	I_1
		18.6	20.1	
				38.7
X-2 5lb X-2 2" X-2 0.2" X-2 0.03"	"	~~18.5~~	23.1	I_2
		18.5	23.1	
				41.6
"	"	19.7	26.8	I_3
		19.8	26.7	
+100 mlb				46.6
Yoke Assy	"	19.8	23.9	I_4
		19.9	23.8	
+100 mlb				43.8

Yoke Assy - Stds
- 2.9
- 2.8
$-2.85 \times \frac{100.0}{5.0} = -57$ mlb

Cr (X-2 5lb) = + 42 "
Cr (X-2 2") = + 24 "
Cr (X-2 0.2") = + 3 "
Cr (X-2 0.03") = 0 "

Yoke Assy - 7.23 lb = + 12 "

Yoke Assy = 7.230012 lb

Double substitution weighing, with equal arm balance, illustrating application of the correction for standard.

FIGURE 9

Temperature 24.8° C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS		Sheet
Humidity 57 %	SUBSTITUTION WEIGHING		Unit (Wts.)
Barometer 749.6 mm	Single Pan Damped Balances		Unit (Cr. & Dill.)
ρ = 1.16	OBSERVATION SHEET		

Observer PCM | Balance M-6 | Date 8-9-66 | Set | NBS Test No. 187910

Load	Dial Setting	Scale Reading	Computations
Wt # 8	5.0	I_1 31.97	I_1 Wt # 8 − Std = +29.24 +29.26 Mean +29.25 × 500.1/50.03 = 292.5 mlb
X-2 5lb	"	I_2 2.73	Cr. Std 5lb = +41.7 "
" +500 mlb	"	I_3 52.76	PΔV = +15.6 "
Wt # 8 +500 mlb	"	I_4 82.02	Wt # 8 = 5lb + 352.5 mlb
			a
+ Vol. Wt # 8 = +		290.78 cm³	
− Vol Std (5lb) = −		283.50 "	
ΔV = +		7.28 cm³	
PΔV = +		8.45 mg	
X		2.2	
		18.6 mlb	a
			a
			a

Single substitution weighing, with one-pan balance, illustrating the application of the buoyancy correction.

FIGURE 10

Temperature 23.1 °C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet 2
Humidity 52 %	SUBSTITUTION WEIGHING Single Pan Damped Balances	Unit (Wts.)
Barometer 755.79 mm	OBSERVATION SHEET	Unit (Cr. & Dill.)
p = 1.18		

Observer PCM | Balance M-4 | Date 9-17-64 | Set B | NBS Test No. 177064

Load	Dial Setting	Scale Reading	Computations
Wt. # B3-130 TW	130.6	I_1 24.93	(B3-160) − Std. = 8.99
TW 0	0.0	I_2 15.94	8.97 / 8.98 × 20.01/19.95 = 9.01 mg
TW " +L20 mg	0.0	I_3 35.89	Cr. for Built-in Weights: Dial Setting 100 = +1.18; " " " 30 = +0.37; " " " 0.6 = +0.02 = 1.57 "
Wt # B3-130 TW +L20 mg	130.6	I_4 44.86	PΔV = −0.10 "
			Wt # B3-160 = 130.6 g + 10.48 mg
	.	.	
	.	.	
	.	.	
	.	.	
Vol of Wt. B3-160	. =	16.745	cm³
− Vol of Built-in Wts. (130.6g) =		−16.829	"
Δ V	. =	−0.084	cm³
	.	.	
P ΔV = 1.18 ×	0.084 =	−0.10 mg	
	.	.	
	.	.	
	.	.	
	.	.	
	.	.	
	.	.	

Double substitution weighing, using the balance's built-in weights as standards, illustrating the application of the correction for the standard and the application of the buoyancy correction.

FIGURE 11

Temperature 24.7° C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS		Sheet
Humidity 56 %	SUBSTITUTION WEIGHING Single Pan Damped Balances OBSERVATION SHEET		Unit (Wts.)
Barometer 750.1 mm			Unit (Cr. & Dill.)
ρ = 1.16			

Observer JHF	Balance M-6	Date 8-10-66	Set C	NBS Test No. 87910

	Load	Dial Setting	Scale Reading	Computations
a	Wt # 132	2.2	14.67 I_1	Wt #132 - Std = +14.46 +14.40 +14.43 × 500.1/50.03 = +144.2 mlb cr X-2 2 lb = + 24.2 " cr X-2 0.2 lb = + 3.2 " ρΔV = + 8.2 " +2.2 lb = 2.200000.0 Wt #132 = 2.200179.8 lb
	X-2 2 lb X-2 0.2 lb	"	0.21 I_2	
	" +500 mlb	"	50.24 I_3	
	Wt # 132 +500 mlb	"	64.64 I_4	
b	Wt # 132 *	2.2	14.72	Wt. #133 - Wt #132 = +10.08 +10.03 +10.06 × 500.1/50.03 = + 100.6 mlb Value #132 = 2.200180.2 lb below #133 = 2.200280.8 lb
	Wt # 133	"	24.80	
	" +500 mlb	"	74.83	
	Wt # 132 +500 mlb	"	64.80	
	* Weight #132 is duplicate standard.			
c	Wt # 132	2.2	14.70	Wt #132 - Std. +14.48 +14.52 +14.50 × 500.1/50.05 = +144.9 mlb cr X-2 2 lb = + 24.2 " cr X-2 0.2 lb = + 3.2 " ρΔV = + 8.2 " +2.2 lb 2.200000.0 Wt #132 = 2.200180.5 lb " " = 2.200179.8 lb. from above Obs. Mean Value #132 = 2.200180.2 lb.
	X-2 2 lb X-2 0.2 lb	"	0.22	
	" +500 mlb	"	20.27	
	Wt # 132 +500 mlb	"	64.79	
	"	"		

Illustrating use of "transfer standard" (see Page 37)

Figure 11 - Illustrating use of "transfer standard"

(a) First calibration of "transfer standard" (Weight No. 132) using known weights;

(b) Calibration of other weights of the same denomination as the "transfer standard" using the "transfer standard" as the standard.

(c) The repeat calibration of the "transfer standard" after all the other weights of the same denomination have been tested. The mean of the two values (a) and (c) found for the "transfer standard", Weight No. 132, is the value of the weights calibrated using the "transfer standard".

FIGURE 12

Temperature 24.6°C	FORM NBS-345.06 (6-1-61) U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS	Sheet
Humidity 46 %	SUBSTITUTION WEIGHING Single Pan Damped Balances OBSERVATION SHEET	Unit (Wts.)
Barometer 750.6 mm		Unit (Cr. & Dill.)
ρ = 1.16 mg/cm³		

Observer R. M.	Balance M-6	Date 10-11-67	Set	NBS Test No. 168493

Load	Dial Setting	Scale Reading	Computations
Wt #10 T 0.05	2.00	14.67	wt #10 + T0.05 − [illegible] = ... a + 2.36 + 2.40 + 2.38 × 500.1/50.08 = +23.8 mlb + Cw of T2 lb = +23.2 " − Cw of T0.05 lb = − 3.1 " ρΔV = + 7.0 " wt #10 = 2 lb − 0.05 lb +50.9 mlb
T 2 lb	"	12.31	
T 2 lb + 500 mlb	"	62.39	
Wt #10 T 0.05 lb + 500 mlb	"	64.79	
	.	.	wt #10 = 1.95 lb +51 mlb. a
	.	.	
+ Vol Wt #10 =	113.23	cm³	
+ Vol T 0.05 lb =	2.84	"	
− Vol T 2 lb	−113.35	.	a
ΔV =	+2.72	cm³	
ρ ΔV =	.	3.16 mg	
	X .	2.2	
	.	7.0 mlb	a
	.	.	
	.	.	
	.	.	
	.	.	a
	.	.	
	.	.	

Double substitution weighing, with one-pan balance, illustrating the computation of the mass value when added weights are used. The application of the corrections for the added weight, the standard, and the buoyancy correction.

FIGURE 13

TEMPERATURE 24.1 HUMIDITY 44% PRESSURE 749.2 TEST NO. 188109 SET A
OBSERVER R.M.B. BALANCE S-1 DATE 10-9-67 SHEET 4 SERIES 3
PAN LOAD

11-10-65

LEFT	RIGHT			
20lb$_1$ T1000mlb T300mlb	CW	~~11.0~~	14.9	
		11.2	14.7	
				26.0
20lb$_2$	"	~~8.6~~	11.0	
		8.8	11.0	
				19.8
a				
20lb$_2$ + 500mlb	"	~~+3.0~~	16.3	
		13.2	16.1	
				29.4
20lb$_1$ T1000mlb T300mlb + 500mlb	"	~~+6.8~~	19.0	
		16.9	18.8	
				35.8
20lb$_1$	CW	~~12.0~~	17.3	
		12.2	17.0	
				29.4
20lb$_2$	CW T1000mlb T300mlb	~~10.1~~	13.2	
		10.2	13.0	
				23.3
b				
20lb$_2$ + 500mlb	CW T1000mlb T300mlb	~~14.2~~	18.5	
		14.4	18.3	
				32.8
20lb$_1$ + 500mlb	CW	17.1	~~22.0~~	
		17.3	21.8	
				39.2

$$20lb_1 - 20lb_2$$
$$\frac{26.0-19.8+35.8-29.4}{2} \times \frac{500.1}{29.4-19.8}$$
$$= \frac{(+6.2+6.4)}{2} \times \frac{500.1}{9.6}$$
$$= +6.3 \times \frac{500.1}{9.6} = +328 \text{ mlb}$$
$$-T1000\text{mlb} = -1000 \text{ "}$$
$$-T300\text{mlb} = -300 \text{ "}$$
$$+Cr\ 20lb_2 = +60 \text{ "}$$
$$20lb_1 = 20\text{lb} - 912 \text{ mlb}$$

$$20lb_1 - 20lb_2$$
$$\frac{+6.1+6.4}{2} \times \frac{500.1}{9.5} = +326 \text{ mlb}$$
$$-T1000\text{mlb} = -1000 \text{ "}$$
$$-T300\text{mlb} = -300 \text{ "}$$
$$+Cr\ 20lb_2 = +60 \text{ "}$$
$$20lb_1 = 20\text{lb} - 914 \text{ mlb}$$

FIGURE 13 (Continued)

TEMPERATURE 24.1 HUMIDITY 44% PRESSURE 749.2 TEST NO. 188109 SET A
OBSERVER R.M.B BALANCE S-1 DATE 10-9-67 SHEET 5 SERIES 3
PAN LOAD

11-10-65

LEFT	RIGHT			
		~~9.9~~	13.1	
20 lb$_3$	C W	10.0	13.1	
	T 3000 mlb			
				23.1
		~~12.2~~	13.5	
20 lb$_2$	C W	12.4	13.3	
				25.8
C		~~16.8~~	18.7	
20 lb$_2$	C W	16.9	18.5	
+500 mlb				
				35.5
		~~14.0~~	18.6	
20 lb$_3$	C W	14.2	18.3	
+500 mlb	T 3000 mlb			
				32.6
20 lb$_2$ is "Transfer Standard" previously calibrated. Corrections for added weights are negligible unless shown.				

20 lb$_3$ − 20 lb$_2$

$\frac{-2.7-2.9}{2} \times \frac{500.1}{9.7} = -144$ mlb

+T 3000 lb = +3006 "

or 20 lb$_2$ = + 60 "

20 lb$_3$ = 20 lb + 2922 mlb

Illustrating computation of mass value when added weights are used, double substitution weighing with equal-arm balance using "transfer standard". (a) Weight under test lighter than standard, added weights on pan with weight under test. (b) Weight under test lighter than standard, added weights on pan with counterweight when standard is on the "load pan". (c) Weight under test heavier than standard added weights on pan with counterweight when weight under test is on the "load pan."

FIGURE 14

TEMPERATURE 24.4 HUMIDITY 45% PRESSURE 748.1 TEST NO. 18809 SET A
OBSERVER R.M.B BALANCE S-1 DATE 10-8-67 SHEET 3 SERIES 2
PAN LOAD

11-10-65

	LEFT	RIGHT			
	20 lb 4 T1000 mlb	20 lb 2	~~13.0~~	14.4	
			13.0	14.3	
					27.4
	20 lb 2	20 lb 4 T1000 mlb	~~11.0~~	12.9	
			11.1	13.1	
					24.1
a	20 lb 2 +500 mlb	20 lb 4 T1000 mlb	~~15.2~~	18.1	
			15.4	18.3	
					33.6
	20 lb 4 T2000 mlb	20 lb 2	~~11.6~~	12.7	
			11.8	12.5	
					24.4
b	20 lb 2	20 lb 4	~~9.9~~	11.1	
			10.0	11.0	
					21.0
	20 lb 2 +500 mlb	20 lb 4	~~14.1~~	16.5	
			14.3	16.3	
					30.7
	20 lb 2 is "Transfer Standard", previously calibrated corrections for added weights are negligible.				

a:

$20\,lb_4 - 20\,lb_2$

$= +\frac{27.4 - 24.1}{2} \times \frac{500.1}{33.6 - 24.1}$

$= +1.65 \times \frac{500.1}{9.5} = +87$ mlb = a

$-$ T1000 mlb $= -1000$ "

$+$ Cr $20\,lb_2 = +60$ "

$20\,lb_4 = 20\,lb - 853$ mlb.

b:

$20\,lb_4 - 20\,lb_2$

$= +\frac{24.4 - 21.0}{2} \times \frac{500.1}{30.7 - 21.0}$

$= +1.7 \times \frac{500.1}{9.7} = +88$ mlb.

$-\frac{1}{2}$(T2000 mlb) $= -1000$ "

$+$ Cr $20\,lb_2 = +60$ "

$20\,lb_4 = 20\,lb - 852$ mlb.

Illustrating computation of mass value when added weights are used; single transposition weighing using "transfer standard". Weight under test lighter than standard. (a) Added weights on pan with weight under test during entire weighing. (b) Added weights on pan with weight under test during half of transposition.

APPENDIX

TABLE I

Magnitude of error introduced by considering the true mass and apparent mass vs brass to be the same for various ranges of densities.

Magnitude of Error	Density Range
1 part in 10^5	7.85 g/cm^3 to 9.00 g/cm^3
2 parts in 10^5	7.35 g/cm^3 to 9.75 g/cm^3
3 parts in 10^5	6.94 g/cm^3 to 10.6 g/cm^3
4 parts in 10^5	6.56 g/cm^3 to 11.62 g/cm^3
5 parts in 10^5	6.23 g/cm^3 to 12.86 g/cm^3
1 part in 10^4	4.95 g/cm^3 to 27.63 g/cm^3
3 parts in 10^4	2.7 g/cm^3

TABLE II

In Tables IIa and IIb the temperature range is from 15°C to 35°C increments of 0.5°C. The relative humidity range is from 10% to 90% in increments of 5%. The barometric pressure range is from 575mm to 780mm in increments of 5mm. The barometric reduction factor to the barometric pressure for atmospheric humidity is given in the body of Table IIa for various temperatures and relative humidities. The air density, in milligrams per cubic centimeter, for the stated conditions is given in the body of Table IIb to 0.01mg/cm^3. To get the air density in pounds per cubic foot, multiply the air density in mg/cm^3 by 0.06242788; and to get the air density in pounds per cubic inch multiply the air density in mg/cm^3 by 0.000036127.

The following procedure is used to find the air density, ρ, with Tables IIa and IIb.

Enter Table IIa through the temperature and relative humidity. In the column headed Air Temp °C of Table IIa, find the temperature nearest the thermometer reading, then go across the table to the column under the relative humidity nearest the relative humidity reading. The number in the body of the table at the intersection of the relative humidity column and the temperature line is the reduction factor and is subtracted from the barometer reading. The resulting figure is the reduced barometer reading and is used to enter Table IIb. In the column headed Air Temp °C of Table IIb, find the temperature nearest the thermometer reading, then go across the table to the column under the barometric pressure nearest the reduced barometer reading. The figure in the body of Table IIb at the intersection of the barometric pressure column and the temperature line is the density of the air. The following example illustrates the procedure for using the tables.

Find the air density (ρ) when:

The temperature is 24.8° C
The relative humidity is 57%
The barometric pressure is 749.6mmHg.

Step 1. In Table IIa the temperature nearest 24.8 °C is 25.0 °C, go along the line to the column under relative humidity nearest 57%, which is the 55% column. The number at the intersection of the 25°C line and the 55% column is 5.0.

Step 2. Subtract 5.0 from the barometer reading, 749.6. 749.6mm - 5.0 = 744.6mm. This is the reduced barometric pressure.

Step 3. In Table IIb, find the 25.0°C line (the temperature in the table nearest 24.8 °C) across the table to the column under barometer pressure nearest 744.6mm, which is the 745mm column. The number at the intersection of the 25°C line and 745mm column, 1.16, is the air density, ρ, in milligrams per cubic centimeter, for the stated conditions.

The air density found, 1.16 mg/cm^3, in lb/ft^3 is

$$1.16 \times 0.06242788 = 0.07241634 \text{ lb/ft}^3$$

in $lb/in.^3$ is

$$1.16 \times 0.000036127 = 0.00041907 \text{ lb/in}^3$$

TABLE IIa

REDUCTION OF BAROMETRIC HEIGHT FOR ATMOSPHERIC HUMIDITY (MILLIMETERS OF MERCURY)

AIR TEMP	RELATIVE HUMIDITY OR PERCENTAGE OF SATURATION																
C	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
15.0	.4	.6	.7	.9	1.1	1.3	1.5	1.7	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.3
15.5	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6
16.0	.4	.6	.8	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.8	3.0	3.2	3.4	3.6	3.8
16.5	.5	.7	.9	1.1	1.4	1.6	1.8	2.0	2.3	2.5	2.7	2.9	3.2	3.4	3.6	3.8	4.1
17.0	.5	.7	1.0	1.2	1.4	1.7	1.9	2.2	2.4	2.6	2.9	3.1	3.4	3.6	3.8	4.1	4.3
17.5	.5	.8	1.0	1.3	1.5	1.8	2.0	2.3	2.5	2.8	3.0	3.3	3.5	3.8	4.0	4.3	4.6
18.0	.5	.8	1.1	1.3	1.6	1.9	2.1	2.4	2.7	2.9	3.2	3.5	3.7	4.0	4.3	4.5	4.8
18.5	.6	.8	1.1	1.4	1.7	2.0	2.2	2.5	2.8	3.1	3.4	3.6	3.9	4.2	4.5	4.8	5.0
19.0	.6	.9	1.2	1.5	1.8	2.1	2.4	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.7	5.0	5.3
19.5	.6	.9	1.2	1.5	1.8	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.6	4.9	5.2	5.5
20.0	.6	1.0	1.3	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.9	4.2	4.5	4.8	5.1	5.5	5.8
20.5	.7	1.0	1.3	1.7	2.0	2.3	2.7	3.0	3.3	3.7	4.0	4.3	4.7	5.0	5.4	5.7	6.0
21.0	.7	1.0	1.4	1.7	2.1	2.4	2.8	3.1	3.5	3.8	4.2	4.5	4.9	5.2	5.6	5.9	6.3
21.5	.7	1.1	1.4	1.8	2.2	2.5	2.9	3.3	3.6	4.0	4.3	4.7	5.1	5.4	5.8	6.1	6.5
22.0	.8	1.1	1.5	1.9	2.3	2.6	3.0	3.4	3.8	4.1	4.5	4.9	5.3	5.6	6.0	6.4	6.8
22.5	.8	1.2	1.6	1.9	2.3	2.7	3.1	3.5	3.9	4.3	4.7	5.1	5.4	5.8	6.2	6.6	7.0
23.0	.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0	6.4	6.8	7.2
23.5	.8	1.2	1.7	2.1	2.5	2.9	3.3	3.7	4.2	4.6	5.0	5.4	5.8	6.2	6.7	7.1	7.5
24.0	.9	1.3	1.7	2.1	2.6	3.0	3.4	3.9	4.3	4.7	5.2	5.6	6.0	6.4	6.9	7.3	7.7
24.5	.9	1.3	1.8	2.2	2.7	3.1	3.5	4.0	4.4	4.9	5.3	5.8	6.2	6.6	7.1	7.5	8.0
25.0	.9	1.4	1.8	2.3	2.7	3.2	3.7	4.1	4.6	5.0	5.5	5.9	6.4	6.8	7.3	7.8	8.2
25.5	.9	1.4	1.9	2.4	2.8	3.3	3.8	4.2	4.7	5.2	5.6	6.1	6.6	7.1	7.5	8.0	8.5
26.0	1.0	1.5	1.9	2.4	2.9	3.4	3.9	4.4	4.8	5.3	5.8	6.3	6.8	7.3	7.7	8.2	8.7
26.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
27.0	1.0	1.5	2.0	2.6	3.1	3.6	4.1	4.6	5.1	5.6	6.1	6.6	7.2	7.7	8.2	8.7	9.2
27.5	1.0	1.6	2.1	2.6	3.1	3.7	4.2	4.7	5.2	5.8	6.3	6.8	7.3	7.9	8.4	8.9	9.4
28.0	1.1	1.6	2.2	2.7	3.2	3.8	4.3	4.8	5.4	5.9	6.5	7.0	7.5	8.1	8.6	9.1	9.7
28.5	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.5	6.1	6.6	7.2	7.7	8.3	8.8	9.4	9.9
29.0	1.1	1.7	2.3	2.8	3.4	4.0	4.5	5.1	5.7	6.2	6.8	7.3	7.9	8.5	9.0	9.6	10.2
29.5	1.2	1.7	2.3	2.9	3.5	4.1	4.6	5.2	5.8	6.4	6.9	7.5	8.1	8.7	9.3	9.8	10.4
30.0	1.2	1.8	2.4	3.0	3.6	4.1	4.7	5.3	5.9	6.5	7.1	7.7	8.3	8.9	9.5	10.1	10.7
30.5	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.5	6.1	6.7	7.3	7.9	8.5	9.1	9.7	10.3	10.9
31.0	1.2	1.9	2.5	3.1	3.7	4.3	5.0	5.6	6.2	6.8	7.4	8.1	8.7	9.3	9.9	10.5	11.1
31.5	1.3	1.9	2.5	3.2	3.8	4.4	5.1	5.7	6.3	7.0	7.6	8.2	8.9	9.5	10.1	10.8	11.4
32.0	1.3	1.9	2.6	3.2	3.9	4.5	5.2	5.8	6.5	7.1	7.8	8.4	9.0	9.7	10.3	11.0	11.6
32.5	1.3	2.0	2.6	3.3	4.0	4.6	5.3	5.9	6.6	7.3	7.9	8.6	9.2	9.9	10.6	11.2	11.9
33.0	1.3	2.0	2.7	3.4	4.0	4.7	5.4	6.1	6.7	7.4	8.1	8.8	9.4	10.1	10.8	11.4	12.1
33.5	1.4	2.1	2.7	3.4	4.1	4.8	5.5	6.2	6.9	7.6	8.2	8.9	9.6	10.3	11.0	11.7	12.4
34.0	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7.0	7.7	8.4	9.1	9.8	10.5	11.2	11.9	12.6
34.5	1.4	2.1	2.9	3.6	4.3	5.0	5.7	6.4	7.1	7.9	8.6	9.3	10.0	10.7	11.4	12.1	12.9
35.0	1.5	2.2	2.9	3.6	4.4	5.1	5.8	6.5	7.3	8.0	8.7	9.5	10.2	10.9	11.6	12.4	13.1

TABLE IIb

DENSITY OF DRY AIR (MILLIGRAMS PER CUBIC CENTIMETER)

AIR TEMP	PRESSURE IN MILLIMETERS OF MERCURY (0C, STANDARD GRAVITY)																			
C	575	580	585	590	595	600	605	610	615	620	625	630	635	640	645	650	655	660	665	670	675
15.0	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07	1.08	1.09
15.5	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.05	1.05	1.06	1.07	1.08	1.09
16.0	.92	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.08	1.08
16.5	.92	.93	.94	.95	.95	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.03	1.03	1.04	1.05	1.06	1.07	1.07	1.08
17.0	.92	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07	1.08
17.5	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.05	1.05	1.06	1.07	1.08
18.0	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.05	1.05	1.06	1.07	1.08
18.5	.92	.92	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.08
19.0	.91	.92	.93	.94	.95	.95	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.03	1.03	1.04	1.05	1.06	1.07	1.07
19.5	.91	.92	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06	1.07
20.0	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.05	1.05	1.06	1.07
20.5	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07
21.0	.91	.92	.92	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.03	1.03	1.04	1.05	1.06	1.07
21.5	.91	.91	.92	.93	.94	.95	.95	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06	1.06
22.0	.90	.91	.92	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.05	1.06
22.5	.90	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.04	1.05	1.06
23.0	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06
23.5	.90	.91	.92	.92	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.03	1.03	1.04	1.05	1.06
24.0	.90	.91	.91	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.06
24.5	.90	.91	.91	.92	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.05	1.05
25.0	.90	.90	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04	1.05
25.5	.89	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.03	1.04	1.05
26.0	.89	.90	.91	.92	.92	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05
26.5	.89	.90	.91	.91	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.02	1.02	1.03	1.04	1.05
27.0	.89	.90	.91	.91	.92	.93	.94	.94	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04	1.04
27.5	.89	.90	.90	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.04	1.04
28.0	.89	.89	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.03	1.03	1.04
28.5	.89	.89	.90	.91	.92	.92	.93	.94	.95	.95	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02	1.03	1.04
29.0	.88	.89	.90	.91	.91	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04
29.5	.88	.89	.90	.91	.91	.92	.93	.94	.94	.95	.96	.97	.97	.98	.99	1.00	1.01	1.01	1.02	1.03	1.04
30.0	.88	.89	.90	.90	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02	1.03	1.03
30.5	.88	.89	.89	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02	1.03
31.0	.88	.89	.89	.90	.91	.92	.92	.93	.94	.95	.95	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02	1.03
31.5	.88	.88	.89	.90	.91	.91	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02	1.03
32.0	.88	.88	.89	.90	.91	.91	.92	.93	.94	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02	1.03
32.5	.87	.88	.89	.90	.90	.91	.92	.93	.93	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02	1.03
33.0	.87	.88	.89	.90	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02	1.02
33.5	.87	.88	.89	.89	.90	.91	.92	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02
34.0	.87	.88	.88	.89	.90	.91	.92	.92	.93	.94	.95	.95	.96	.97	.98	.98	.99	1.00	1.01	1.01	1.02
34.5	.87	.88	.88	.89	.90	.91	.91	.92	.93	.94	.94	.95	.96	.97	.97	.98	.99	1.00	1.00	1.01	1.02
35.0	.87	.87	.88	.89	.90	.90	.91	.92	.93	.93	.94	.95	.96	.96	.97	.98	.99	.99	1.00	1.01	1.02

TABLE IIb (Continued)

DENSITY OF DRY AIR (MILLIGRAMS PER CUBIC CENTIMETER) CONTINUED

AIR TEMP	PRESSURE IN MILLIMETERS OF MERCURY (0C, STANDARD GRAVITY)																				
C	680	685	690	695	700	705	710	715	720	725	730	735	740	745	750	755	760	765	770	775	780
15.0	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22	1.23	1.23	1.24	1.25	1.26
15.5	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.22	1.22	1.23	1.24	1.25	1.26
16.0	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22	1.23	1.24	1.25	1.25
16.5	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.23	1.24	1.25
17.0	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22	1.22	1.23	1.24	1.25
17.5	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.21	1.22	1.23	1.24	1.25
18.0	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22	1.23	1.24	1.24
18.5	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.23	1.24
19.0	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22	1.22	1.23	1.24
19.5	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.21	1.22	1.23	1.24
20.0	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22	1.23	1.24
20.5	1.08	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.23
21.0	1.07	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22	1.22	1.23
21.5	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.21	1.22	1.23
22.0	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22	1.23
22.5	1.07	1.08	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22	1.23
23.0	1.07	1.07	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22	1.22
23.5	1.06	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.21	1.22
24.0	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21	1.22
24.5	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21	1.22
25.0	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21	1.22
25.5	1.06	1.07	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.21
26.0	1.06	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20	1.21
26.5	1.05	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20	1.21
27.0	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19	1.20	1.21
27.5	1.05	1.06	1.07	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20	1.21
28.0	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20	1.20
28.5	1.05	1.05	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.19	1.19	1.20
29.0	1.05	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19	1.20
29.5	1.04	1.05	1.06	1.07	1.07	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19	1.20
30.0	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.20
30.5	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19	1.19
31.0	1.04	1.05	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18	1.19
31.5	1.04	1.04	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18	1.19
32.0	1.04	1.04	1.05	1.06	1.07	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19
32.5	1.03	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.16	1.16	1.17	1.18	1.19
33.0	1.03	1.04	1.05	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18	1.18
33.5	1.03	1.04	1.05	1.05	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.11	1.12	1.13	1.14	1.14	1.15	1.16	1.17	1.17	1.18
34.0	1.03	1.04	1.04	1.05	1.06	1.07	1.07	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.13	1.14	1.15	1.16	1.16	1.17	1.18
34.5	1.03	1.03	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.16	1.16	1.17	1.18
35.0	1.03	1.03	1.04	1.05	1.06	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.12	1.12	1.13	1.14	1.15	1.15	1.16	1.17	1.18

TABLE III

CONVERSION TABLE
Some Useful Conversion Factors

To Convert To		From		Multiply By
lb/ft^3	=	mg/cm^3	x	0.062427886
lb/ft^3	=	g/cm^3	x	62.427886
lb/in^3	=	mg/cm^3	x	0.000036127
lb/in^3	=	g/cm^3	x	0.0361272
mg/cm^3	=	lb/ft^3	x	16.018465
mg/cm^3	=	lb/in^3	x	27679.9028
g/cm^3	=	lb/ft^3	x	0.016018465
g/cm^3	=	lb/in^3	x	27.679903
grams	=	ounces	x	28.349523125
grams	=	pounds	x	453.59237
kilograms	=	ounces	x	0.028349523
kilograms	=	pounds	x	0.45359237
milligrams	=	pounds	x	453592.37
ounces	=	grams	x	0.03527396
ounces	=	kilograms	x	35.27396
pounds	=	grams	x	0.00220462
pounds	=	kilograms	x	2.204623
pounds	=	milligrams	x	0.0000022046
cu in.	=	cm^3	x	0.06102374
cu ft	=	cm^3	x	0.00003531467
cm^3	=	cu in.	x	16.387064
cm^3	=	cu ft	x	28316.846592
cm	=	feet	x	30.48
cm	=	inches	x	2.54
feet	=	cm	x	0.0328084
ounces	=	pounds	x	16
pounds	=	ounces	x	0.0625
inches	=	cm	x	0.3937008

TABLE IV

DENSITIES AND VOLUMES PER UNIT MASS
OF
SELECTED MATERIALS

Material	Density at 20°C g/cm^3	Density at 20°C lb/in^3	Volume at 20°C cm^3 Per gram	Volume at 20°C in^3 Per lb
Brass (Normal)	8.3909	0.30314	0.119177	3.29881
Stainless Steel	8.0	0.2890	0.1250	3.4600
Stainless Steel	7.92	0.2861	0.1263	3.4950
Stainless Steel	7.89	0.2850	0.1267	3.5083
Stainless Steel	7.84	0.2832	0.1276	3.5306
Stainless Steel	7.8	0.2818	0.1282	3.5487
Stainless Steel	7.76	0.2803	0.1289	3.5670
Stainless Steel	7.75	0.2800	0.1290	3.5716
Steel	7.83	0.2829	0.1277	3.5351
Cast Iron (Gray)	7.0	0.2529	0.1429	3.9543
Cast Iron (White)	7.6	0.2746	0.1316	3.3948
Nickel Chromium	8.34	0.3013	0.1199	3.3190
Nickel Chromium	8.41	0.3038	0.1189	3.2913
Nickel Chromium	8.5	0.3071	0.1176	3.2565
Tantalum	16.6	0.5997	0.0602	1.6675
Platinum	21.37	0.7720	0.0468	1.2953
Platinum	21.5	0.7767	0.0465	1.2874
Platinum-Iridium	21.54	0.7782	0.0464	1.2850
Aluminum	2.7	0.0975	0.3704	10.2522

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Generally weights for piston gages have odd denominations that are often not readily calibrated by intercomparison methods. Therefore these weights are frequently calibrated by direct comparison methods. This paper presents direct comparison methods for calibrating piston gage weights for use with both equal-arm balances and single-pan balances. Methods of estimating the uncertainty of the values obtained are given. Also included are methods of checking for blunders or gross errors.

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